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GMAT QUANT FORMULA BOOK

All Essential Formulas You Need to Score High



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NUMBER PROPERTIES

A NUMBERS

All numbers tested on the GMAT are real numbers, meaning they lie on the number line. These include:

- **Integers:** whole numbers (... , -2, -1, 0, 1, 2, ...)
- **Rational numbers:** numbers that **can be written as a fraction** (terminating decimals)
- **Irrational numbers:** numbers that **cannot be written as fractions** (non-terminating decimals)

Example:

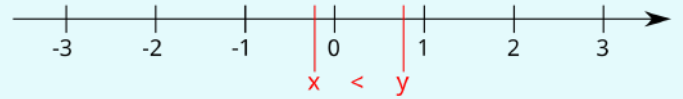
- Rational: $\frac{3}{4}$, 0.25, 0.333...
- Irrational: $\sqrt{2}$, π

GMAT Trap

- Rational $\neq$ integer
- Irrational $\neq$ decimal

NOTE: Decimal terminates if denominator has only 2s and/or 5s

B SIGN RULES



Here, $x < 0 \rightarrow$ negative and $y > 0 \rightarrow$ positive

Sign Rules

- $(+) \times (+) = (+)$
- $(-) \times (-) = (+)$
- $(+) \times (-) = (-)$

Same rules apply for division.

NOTE: Zero: (1) is an integer, (2) is even, (3) is neither positive nor negative, and (4) is divisible by every non-zero integer

C ODD & EVEN

Even number: divisible by 2

- Written as $2k$ ($k \rightarrow$ any integer)

Odd number: not divisible by 2

- Written as $2k + 1$ ($k \rightarrow$ any integer)

Power Rules

- $\text{odd}^n = \text{odd}$
- $\text{even}^n = \text{even}$
- $(\text{odd/even} \pm \text{even/odd})^n \rightarrow$ depends on base parity, not exponent

Example:

- If $k = 3 \rightarrow 2k = 6$ (even), $2k + 1 = 7$ (odd)

Outcome

If a^2 is even $\rightarrow a$ must be even
If a^2 is odd $\rightarrow a$ must be odd

D PRIME NUMBERS

A prime number is a **positive** integer that has exactly **two** positive factors: **1** and **itself**.

Remember:

- 1 is **not prime** | 2 is the only **even prime**
- All primes greater than 3 are of the form $6k \pm 1$

A composite number is an integer greater than 1 that is not prime and has **more than two** factors.

Is a 'n' a prime number?

If a positive integer n has any factor other than 1 and itself, then at least one of those factors must be less than or equal to $\sqrt{n} \rightarrow$ That's why: we only need to test prime numbers $\leq \sqrt{n}$, not all numbers.

E = Even | O = Odd

E	±	E	→	E
E	±	O	→	O
O	±	O	→	E
E	×	E	→	E
E	×	O	→	E
O	×	O	→	O

1

Compute $\sqrt{n}$

Example: $n = 161 \mid \sqrt{161} \approx 12.7$

2

List all prime numbers $\leq \sqrt{n}$

Example: Primes $\leq 12.7 \rightarrow 2, 3, 5, 7, 11$

3

Divide n by each prime from previous step: if any division is exact $\rightarrow n$ is not prime

Example:

- $161 \div 2 \times \mid 161 \div 3 \times$
- $161 \div 5 \times \mid 161 \div 7 = 23 \checkmark$

! 161 is **NOT** prime

Prime numbers below 100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

NOTES

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E

PRIME FACTORISATION

Every integer 'n' greater than 1 can be written uniquely as a product of prime numbers ($p_1, p_2, \dots$):

$$n = p_1^a \times p_2^b \times \dots$$

The **total number of factors** (f) in this case:

$$f = (a + 1)(b + 1) \dots$$

★ **Example:**

$$450 = 2^1 \times 3^2 \times 5^2$$

$$\text{Number of factors} = (1+1)(2+1)(2+1) = 18$$

⚠ **GMAT Trap**

- Forgetting to include 1
- Perfect squares → odd number of factors

The **sum of factors** (S) in this case:

$$S = \frac{(p_1^{a+1} - 1)}{(p_1 - 1)} \times \frac{(p_2^{b+1} - 1)}{(p_2 - 1)} \times \dots$$

The number of odd (O) and even factors (E) of 'n', such that:

$$n = 2^a \times p_2^b \times p_3^c \times \dots$$

- Number of **odd** factors (O) = $(b + 1) \times (c + 1) \times \dots$
- Number of **even** factors (E) = $(a) \times (b + 1) \times (c + 1) \times \dots$

If 'n' has 'm' distinct prime factors, then the number of coprime factor pairs (k):

$$k = 2^{(m-1)}$$

The total number of ways to write 'n' as product of two factors, if 'n' has 'k' total factors:

- If k = even → $k/2$ ways
- If k = odd → $(k + 1)/2$ ways

🔑 **NOTE:** Proper factors are all factors except the number itself.

A number whose proper factors add up to the number is called a perfect number (ex: 6)

F

DIVISIBILITY & REMAINDERS

➡ Memorise divisibility rules to reduce calculation time

When numbers are raised to powers, their last digits repeat in fixed cycles, called **cyclicity**.

1. Powers of numbers ending in 0, 1, 5, or 6 always have the same last digit.
2. Powers of numbers ending in 4 or 9 repeat every **2** powers.
3. Powers of numbers ending in 2, 3, 7, or 8 repeat every **4** powers.

Number 'n', divisor 'd', leaving a quotient 'q' and remainder 'r':

$$n = (d)(q) + r$$

⚡ If 'a' and 'b' are two numbers such that $a \equiv b \pmod{m}$, then:

- $a + c \equiv b + c \pmod{m}$
- $a \times c \equiv b \times c \pmod{m}$
- $a^k \equiv b^k \pmod{m}$

⚡ Addition & Multiplication of remainders

- $\text{Rem}[(x + y + z) \div d] = \text{Rem}[x \div d] + \text{Rem}[y \div d] + \text{Rem}[z \div d]$
 - $\text{Rem}[(z \times y \times x) \div d] = \text{Rem}[x \div d] \times \text{Rem}[y \div d] \times \text{Rem}[z \div d]$
- If the **sum > d**, then $\text{Rem} = \text{Rem}[\text{sum} \div d]$

★ **Example:**

$$\text{Rem}[(17 \times 19) \div 6] = (5 \times 1) = 5$$

⚠ Always reduce each term first

★ **Example:**

$$\begin{aligned} 6^{30} &\equiv 1 \pmod{31} \\ 6^{31} &\equiv 6 \pmod{31} \end{aligned}$$

🔑 **Fermat's Theorem**

If 'n' is prime and if 'm' is not divisible by 'n', then:

- $m^{n-1} \equiv 1 \pmod{n}$
 - $m^n \equiv m \pmod{n}$
- ⚠ This only works if divisor is **prime**

⚡ If, when divided by divisor 'd':

- x leaves remainder **p**
- y leaves remainder **q**
- z leaves remainder **r**

Then:

- d divides **(x - p), (y - q), (z - r)**

⚡ If x, y, z all leave the same remainder 'r' when divided by divisors **d₁, d₂, d₃**, then:

$$\bullet \mathbf{x = LCM(d_1, d_2, d_3) + r}$$

⚡ Algebraic expressions:

- $(a^n + b^n)$ is divisible by $(a + b)$, when n is **odd**.
- $(a^n - b^n)$ is divisible by $(a + b)$, when n is **even**.
- $(a^n - b^n)$ is always divisible by $(a - b)$, for every n.

N	Cyclicity	Rule
2	2, 4, 6, 8	Last digit is 0, 2, 4, 6, 8
3	3, 9, 7, 1	Sum of digits divisible by 3
4	4, 6	Last two digits divisible by 4
5	5	Ends in 0 or 5
6	6	Divisible by both 2 and 3
7	7, 9, 3, 1	Remove last digit, double it, subtract from remaining number; repeat until manageable
8	8, 4, 2, 6	Last three digits divisible by 8
9	9, 1	Sum of digits divisible by 9
10	0	Ends in 0
11	1	(Sum of odd-position digits - sum of even-position digits) divisible by 11

NOTES



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G

GCF-LCM & SQUARES

- GCF: product of common primes with the smallest powers
 - LCM: product of all primes with the largest powers
- If two numbers share no common primes, GCF = 1

Product of two numbers equals product of their GCF and LCM:

$$a \times b = GCF(a, b) \times LCM(a, b)$$

💡 To find the LCM of two fractions f_1 and f_2 :

$$LCM(f_1, f_2) = \frac{LCM(\text{numerators})}{HCF(\text{denominators})}$$

💡 A number is a **perfect square** if and only if all prime exponents are even. Consequences:

- **Odd** number of total factors
- Square roots are integers
- The sum of factors is **odd**

⚠️ **NOTE:** Squares can be represented as:
 $3k$ or $3k + 1$ ($k \rightarrow$ any +integer)
 $4k$ or $4k + 1$ ($k \rightarrow$ any +integer)

💡 A number is a **perfect cube** if and only if all prime exponents are multiples of three.

💡 A perfect square can only end in: **{0, 1, 4, 5, 6, 9}**

- Second-last digit is always **even**, except when last digit = 6
- If last digit = 5 $\rightarrow$ second-last digit must be 2

H

FACTORIALS

Product of **n** consecutive integers divisible by **n!**:

$$n! = n \times (n-1) \times (n-2) \times \dots \times 1$$

💡 **Example:**

$2! = 2, 3! = 6, 4! = 24, 5! = 120, 6! = 720, 7! = 5040, \dots$

⚠️ **NOTE:**

- If n is odd $\rightarrow$ sum divisible by n
- If n is even $\rightarrow$ sum NOT divisible by n
- $0! = 1$ and negative factorials are undefined

💡 The number of trailing zeroes (T) in $n!$:

$$T = \left[\frac{n}{5} \right] + \left[\frac{n}{25} \right] + \left[\frac{n}{125} \right] + \dots$$

where $[x]$ = greatest integer less than or equal to the given number

💡 The highest power (**H**) of a prime number 'p' in $n!$:

$$H = \left[\frac{n}{p} \right] + \left[\frac{n}{p^2} \right] + \left[\frac{n}{p^3} \right] + \dots$$

where $[x]$ = greatest integer less than or equal to the given number

💡 The product of 'n' consecutive natural numbers is always divisible by $n!$ (💡 **Example:** $5 \times 6 \times 7$ div. by $3!$)

Cubes		Squares			
1^3	1	1^2	1	11^2	121
2^3	8	2^2	4	12^2	144
3^3	27	3^2	9	13^2	169
4^3	64	4^2	16	14^2	196
5^3	125	5^2	25	15^2	225
6^3	216	6^2	36	16^2	256
7^3	343	7^2	49	17^2	289
8^3	512	8^2	64	18^2	324
9^3	729	9^2	81	19^2	361
10^3	1000	10^2	100	20^2	400

NOTES 

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FRACTIONS & DECIMALS | RATIO & PROPORTION | PERCENTS

A FRACTIONS

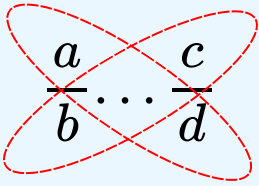
- Fraction = numerator / denominator
- Proper**: numerator < denominator
- Improper**: numerator ≥ denominator
- Mixed number** = integer + proper fraction

Mixed → Improper:

$$a\frac{b}{c} = \frac{ac + b}{c}$$

→ Comparing fractions [Shortcut: **Bow-tie method**]

$$(a \times d) \quad (b \times c)$$



IF: $(a \times d) > (b \times c) \rightarrow \frac{a}{b} > \frac{c}{d}$

IF: $(a \times d) < (b \times c) \rightarrow \frac{a}{b} < \frac{c}{d}$

B DECIMALS

Terminating Decimal

1. Remove decimal
2. Divide by power of 10
3. Reduce

Example:

$$0.56 = \frac{56}{100} = \frac{14}{25}$$

Repeating Decimal

If $x = 0.ab$ (repeating), then:

$$x = \frac{ab}{99}$$

Example:

$$0.39 = \frac{39}{99} = \frac{13}{33}$$

Mixed Repeating Decimal [Rare]

These are of the format: $x = 0.abcdcd\ldots$ (with cd repeating)

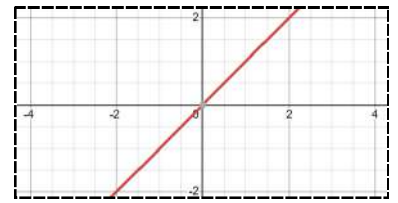
C VARIATION

Direct Variation

$$y \propto x \Rightarrow y = kx$$

Here:

- k = constant
- If x doubles $\rightarrow y$ **doubles**



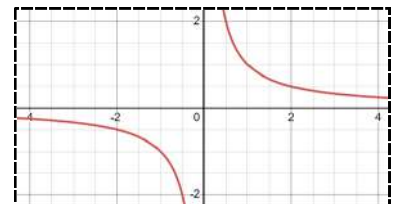
when $k = 1$

Inverse Variation

$$y \propto \frac{1}{x} \Rightarrow y = \frac{k}{x}$$

Here:

- k = constant
- If x doubles $\rightarrow y$ **halves**



when $k = 1$

Combined Variation

$$y \propto \frac{xz}{w} \Rightarrow y = \frac{kxz}{w}$$

Here:

- k = constant
- y, x, z and w = variables

Approach:

1. Write proportionality equation
2. Introduce constant k
3. Use given scenario to find k
4. Apply to new scenario and solve for variables

NOTES



1 Write the full number without decimal up to one full repeat

Example: $n = 0.25121212\ldots$, then write $x = 2512$

Here:

- Non-repeating part = 25
- Repeating part = 12 (2 digits)

2 Subtract the non-repeating part

Example: $x = 2512 - 25 = 2487$

3 In denominator, use 9s for repeating, 0s for non-repeating

Example:

Here -

- 2 repeating digits $\rightarrow 99$
 - 2 non-repeating digits $\rightarrow 00$
- Denominator $\rightarrow 9900$

$$0.251212\ldots = \frac{2487}{9900} = \frac{829}{3300}$$

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D RATIO & PROPORTION

A ratio compares quantities relative to each other, not in absolute terms:

$$a : b = \frac{a}{b}$$

Other representations:

- $a = kb, b = k^{-1}a$
- $a = xt, b = yt$ (scaling variable)

Trap: Assuming actual values without a total or constraint

Approach

If $a : b : c = 2 : 3 : 5$, and if total = T , then:

$$a = 2x, b = 3x, c = 5x$$

such that:

$$x = \frac{T}{2 + 3 + 5}$$

Combining Ratios:

If $a : b = 2 : 3$ and $b : c = 4 : 5$, then to combine the ratios, make the common variable 'b' equal:

- multiply $a : b$ by 4 $\rightarrow a : b = 8 : 12$
- multiply $b : c$ by 3 $\rightarrow b : c = 12 : 15$

Therefore - **$a : b : c = 8 : 12 : 15$**

NOTE: If A is $p\%$ of B, then - $A : B = p : 100$

Ratio Properties

If:

$$\frac{a}{b} = \frac{c}{d}$$

1 Invertendo

$$\frac{b}{a} = \frac{d}{c}$$

2 Alternendo

$$\frac{a}{c} = \frac{b}{d}$$

3 Componendo

$$\frac{a+b}{b} = \frac{c+d}{d}$$

4 Dividendo

$$\frac{a-b}{b} = \frac{c-d}{d}$$

5 Comp.-Dividendo

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}$$

An $x\%$ change in quantity Q can be offset by changing the same quantity by $y\%$ in the opposite direction, where:

$$y = \frac{100x}{100 + x}\%$$

E PERCENTAGES

Representation:

$$\text{percent} = \frac{\text{part}}{\text{whole}} \times 100$$

Percentage change:

$$\text{change} = \frac{\text{new} - \text{original}}{\text{original}} \times 100$$

If A is $x\%$ less than B, then:

$$B \text{ is } \frac{x}{100 - x} \times 100\% \text{ more than A}$$

Example:

A is 20% less than B $\rightarrow$
 $A = 0.8B$
 B is 25% more than A

If A is $x\%$ more than B, then:

$$B \text{ is } \frac{x}{100 + x} \times 100\% \text{ less than A}$$

Example:

A is 20% more than B $\rightarrow$
 $A = 1.2B$
 B is 16⅔% less than A

For successive

changes of $+a\%$ and $+b\%$:
Net change = $(1 + a)(1 + b) - 1$

If value changes by $+x\%$ and then $-x\%$:

$$\text{Net change} = -\frac{x^2}{100}\%$$

Tricks

- $x\%$ of $y = y\%$ of $x \rightarrow$ (25% of 80 = 80% of 25 = 20)
- 10% = move decimal one place
- 5% = $\frac{1}{2}$ of one-tenth of number
- Split ugly percents: 37% of 80 $\rightarrow (40\% - 3\%) \times 80$

GMAT Trap

- Repeated increase & decrease always multiplies
- Same absolute change $\neq$ same percent change
- "More than / less than" signals base switch
- Averaging percents is almost always wrong

Fractions

Percents

Decimals

(1/2)	50%	0.5
(1/3)	33.33...%	0.333...
(1/4)	25%	0.25
(1/5)	20%	0.2
(1/6)	16.66...%	0.1666...
(1/7)	$\approx 14.286\%$	0.142857...
(1/8)	12.50%	0.125
(1/9)	11.11...%	0.111...
(1/10)	10%	0.1
(1/11)	$\approx 9.09\%$	0.090909...
(1/20)	5%	0.05

NOTES

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WORD PROBLEMS | ARITHMETIC

A PROFIT & LOSS

- **CP** = Cost Price
- **SP** = Selling Price
- **MP** = Marked (List) Price
- **Profit** = (SP - CP)
- **Loss** = (CP - SP)

➡ **Profit/Loss percent:**

$$\Delta = \left| \frac{SP - CP}{CP} \right| \times 100$$

⚠ **P/L % is always calculated on CP**

💡 **Relationship b/w CP, SP, MP and Discount:** [p → profit] [l → loss]

CP ↔ SP $SP = CP \times \left(1 + \frac{p}{100}\right)$ ↑ $SP = CP \times \left(1 - \frac{l}{100}\right)$ ↓

MP ↔ SP $SP = MP \times \left(1 - \frac{d}{100}\right)$ ↓ $d\% = d_1 + d_2 - \frac{d_1 \times d_2}{100}$

Effective discount when discounts **d₁%** and **d₂%** are applied

💡 **Type I:** Buy 'x' Get 'y' Free
Equivalent discount (**d%**):

$$d\% = \frac{y}{x+y} \times 100$$

💡 **Example:**

Buy 2 get 1 free → d = 33⅓%
Buy 3 get 2 free → d = 40%

💡 **Type II:** Incorrect Weights
If charged for x units but gives y units, then profit (**p%**):

$$p\% = \left(\frac{x-y}{y} \right) \times 100$$

💡 **Example:** Claims 1 kg, but gives 800 g → Profit % = 25%

C MIXTURES

💡 **Weighted Average**

$$\text{Average} = \frac{\sum (\text{quantity} \times \text{value})}{\sum (\text{quantity})}$$

- Quantity = amount/volume/weight of each component
- Value = price, concentration, strength, or percentage

⚠ **Don't average values directly unless quantities are equal**

💡 **Alligation Rule**

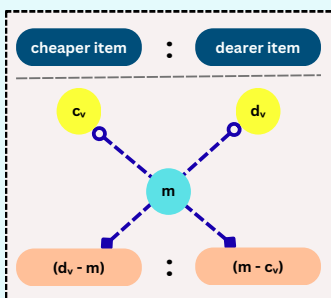
$$\frac{c}{d} = \frac{(d_v - m)}{(m - c_v)}$$

where:

- **c** → cheaper item with lower price/percentage (**c_v**)
- **d** → dearer item with higher price/percentage (**d_v**)
- **m** → mean or final mixture value

⚠ **NOTE:**

- This only works for mixing **TWO** components.
- The component farther from mean contributes **LESS** quantity.



💡 **Successive Dilution**

$$Q_x = Q_o \times \left(1 - \frac{x}{Q_o}\right)^n$$

where:

- **Q_x** → final total volume
- **Q_o** → original total volume
- **x** → volume replaced each time
- **n** → number of replacements

💡 **Example:** If **Q_o** = 10 L milk, **x** = 1 L water, and **n** = 2, then: **Q_x** = 10 × (1 - 1/10)² = 10 × (0.9)² = 8.1 L milk

B INTEREST

- **P** = Principal
- **r%** = Annual rate (%)
- **t** = Time duration
- **A** = Amount
- **SI** = Simple Interest
- **CI** = Compound Interest

➡ **Simple Interest**

$$A = P \left(1 + \frac{r \times t}{100}\right)$$

$$SI = \frac{P \times r \times t}{100}$$

➡ **Compound Interest**

$$A = P \left(1 + \frac{r}{100}\right)^t$$

$$CI = A - P$$

Case Relationship

t = 1	SI = CI
t = 2	CI - SI = P × (r/100) ²
t = 3	CI - SI > P × (r/100) ²

💡 **Compounding frequency:**

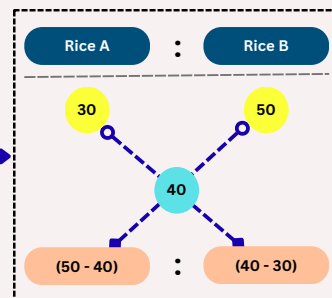
- **Annual** → rate = r, power = t
- **Semi-annual** → rate = (r/2), power = (2t)
- **Quarterly** → rate = (r/4), power = (4t)

💡 An annual installment '**x**' to repay a loan (with principal **P**) with interest rate **r%** per annum over '**t**' years, can be represented as:

$$P = \frac{x}{\left(1 + \frac{r}{100}\right)^t} + \dots + \frac{x}{\left(1 + \frac{r}{100}\right)^2} + \frac{x}{\left(1 + \frac{r}{100}\right)^1}$$

💡 **Example:** A shopkeeper mixes two types of rice: Rice A costing ₹30/kg, and Rice B costing ₹50/kg. He sells the mixture at ₹44/kg and makes a 10% profit. In what ratio is Rice A and Rice B are mixed?

- If profit = 10%, then: **CP** = 44/1.1 = ₹40/kg



∴ Rice A : Rice B = **1 : 1**

💡 **Example:** A loan of ₹12,000 is repaid in 3 equal annual installments at 10% interest p.a. Find the annual installment.

- **P** = 12,000
- **r** = 10%
- **t** = 3
- **Installment** = x

$$\Rightarrow P = [x \div (1.1)] + [x \div (1.1)^2] + [x \div (1.1)^3]$$

$$\Rightarrow 12000 = x(2.486)$$

$$\Rightarrow x = [12000 \div 2.486] \approx \text{₹}4,825$$

NOTES

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Combined Work

$$\text{Work} = \text{Rate} \times \text{Time}$$

If A takes 'x' amount of time and B takes 'y' amount of time individually to complete a piece of work (W), then their combined rate (r) and the combined time (t) to finish the same work is:

$$r = \frac{1}{x} + \frac{1}{y} \quad t = \frac{x \times y}{x + y} \quad r_i = \sum \left(\frac{1}{t_i} \right)$$

Work Done in Segments

$$W = r_1 \times t_1 + r_2 \times t_2 + \dots$$

Track work in fractions; here, the total work (W) is the sum of work done individually

Type I: Workers Added after t

$$T_{\text{total}} = t + \frac{W - (n_o \times r \times t)}{(n_o + n_1) \times r}$$

where:

- T_{total} → total time to finish the work
- W → total work (in work units)
- r → efficiency of one worker (work units per unit time)
- n_o → number of workers initially present
- n_1 → number of workers **added/removed** later
- t → time worked before new workers are **added/removed**

Type II: Workers Removed after t

$$T_{\text{total}} = t + \frac{W - (n_o \times r \times t)}{(n_o - n_1) \times r}$$

Efficiency

If A:B efficiency = m : n, then:

- Time ratio = n : m

This is mainly because **efficiency** ∝ **rate** and **efficiency** ∝ (1/time)

Example: 5 workers start a job; each worker does 2 units/day. They can complete a task in 10 days. If, after 4 days, 3 more workers join, what is the new completion time?

$$T_{\text{total}} = 4 + \frac{100 - (5 \times 2 \times 4)}{(5 + 3) \times 2} = 4 + \frac{60}{16} = 7.75$$

Other Useful Shortcuts:

1. Three People Working Together

If A can complete a job in 'a' hours, B in 'b' hours, and C in 'c' hours, then working together they will complete it in 't' hours, such that:

$$t = \frac{a \times b \times c}{(a \times b + b \times c + c \times a)}$$

2. Two People Working Together

If A can complete a job in 'a' hours, and A and B together can complete it in 't' hours, then B alone will take 'b' hours, such that:

$$b = \frac{(a \times t)}{(a - t)}$$

3. Finding an Individual's Rate When Three Work Together

If A can complete a job in 'a' hours, B in 'b' hours, and A, B, and C together in 't' hours, then:

- C alone can do it in 'c' hours, such that:

$$c = \frac{(a \times b \times t)}{(a \times b - a \times t - b \times t)}$$

- A and C or B and C together can do it in 'T₁' or 'T₂' hours respectively, such that:

$$T_1 = \frac{(b \times t)}{(b - t)} \quad T_2 = \frac{(a \times t)}{(a - t)}$$

Pipes and Cisterns

$$\text{rate} = \underbrace{\sum (\text{inlet})}_{\text{positive}} + \underbrace{\sum (\text{outlet})}_{\text{negative}}$$

Example: Pipe A fills tank in 6 hours. Pipe B empties the same tank in 8 hours. If tank empty + both are opened, the time taken to fill:

Let tank volume = 120 L
inlet rate = 20 L/hr | outlet = 15 L/hr
Net = (20 - 15) = 5 L/hr
Time taken = 120/5 = 24 hrs

Instead of setting total work = 1, set total work = LCM of individual completion times → converts all rates into integers, making multi-step problems far easier.

1

2

3

4



Choose Total Work = LCM of individual times (t₁, t₂, ..., t_n)

Example: A completes work in 10 days, B in 15 days.

- Total work = LCM(10, 15) = 30 units

Compute Individual Efficiencies
efficiency = total work/time taken

Example:

- A's efficiency = 30/10 = 3 units/day
- B's efficiency = 30/15 = 2 units/day

Combine or Modify Efficiencies

- Add efficiencies when working together
- Subtract if someone leaves
- Track work completed in units

Example:

Combined efficiency of A and B = (3 + 2) = 5 units/day

Compute Time = (remaining work)/(net efficiency)

Example:

- Total work = 30 units
- Combined efficiency = 5 units/day
- Time taken for A and B to do the same work together = (30/5) = 6 days



NOTES



Average Speed (s)

Distance = Speed × Time

Case I: Equal Distances

$$s = \frac{2(s_1)(s_2)}{s_1 + s_2}$$

Case II: Equal Time

$$s = \frac{s_1 + s_2}{2}$$

Example:

Two trains 350 m apart move toward each other at 15 and 20 m/s. They'll meet in 10 s.

Case III: Unequal Distance & Time

$$s = \frac{(s_1)(t_1) + (s_2)(t_2)}{t_1 + t_2}$$

NOTE: If speed increases by $x\%$, time decreases by:

$$\frac{x}{100 + x} \times 100\%$$

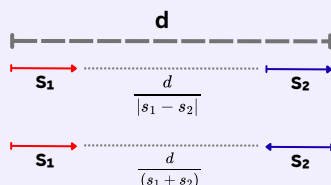
Example:

If speed of a car increases by 20%, the time taken to traverse a distance decreases by $20/120 = \frac{1}{6} = 16.67\%$

Relative Speed (r)

Objects moving in:

- Same direction $\rightarrow |s_1 - s_2|$
- Opposite direction $\rightarrow (s_1 + s_2)$
- Meeting time (T) = (initial distance) / (relative speed)



Speed Ratio and Meeting Time from Post-Meeting Times

A (speed s_a) and B (speed s_b) start from opposite ends toward each other and meet at point P at time t. After meeting:

- A takes t_a to reach B's starting point
- B takes t_b to reach A's starting point

Speeds remain constant throughout

Speed Ratio

$$\frac{s_a}{s_b} = \sqrt{\frac{t_b}{t_a}}$$

Meeting Time

$$t = \sqrt{t_a \times t_b}$$

Example: A takes 4 hrs after meeting, B takes 9 hrs after meeting, then:

$$\frac{s_a}{s_b} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

Conversions

- 1 m/s = **18/5** km/h $\leftrightarrow$ 1 km/h = **5/18** m/s
- 1 mile $\approx$ **1.6** km
- 1 mile = **1760** yards
- 1 yard = **3** ft
- 1 ft = **12** inches
- 1 inch = **2.54** cms

Races and Headstarts

- If A (speed s_a) beats B (speed s_b) by x meters in a L -meter race, then:

$$\frac{s_a}{s_b} = \frac{L}{L - x}$$

- If the same distance is covered and the speed ratio of A : B = $s_a : s_b$, then:

$$\text{Time ratio } \frac{t_a}{t_b} = \frac{s_b}{s_a}$$

- If the speed ratio of A : B = $s_a : s_b$, then the distance covered in equal time:

$$\text{Dist. ratio } \frac{d_a}{d_b} = \frac{s_a}{s_b}$$

- If the B (speed s_b) has 't' seconds head start, then the head start distance (h):

$$h = s_b \times t$$

Example: In a 400-meter race, runner A finishes 40 meters ahead of runner B. In another 400-meter race, runner B is given a head start of 8 seconds and both runners run at their original constant speeds. If runner A just manages to catch runner B exactly at the finish line of the 400-meter track, what is the speed of runner B in meters per second?

- Speed ratio from first race = $400:360 = 10:9$
- B's head start distance = $s_b \times 8$
- A covers 400 m in the same time B covers $[400 - (s_b \times 8)]$ m
- Considering speed ratio:

$$\frac{400}{400 - (s_b \times 8)} = \frac{10}{9} \Rightarrow 400 \times 9 = 10 \times [400 - (s_b \times 8)] \Rightarrow 3600 = 4000 - 80s_b \Rightarrow s_b = 5 \text{ m/s}$$

Trains (Length, Speed) $\rightarrow$ $[L_1, s_1]$ Train A $\cdot$ $[L_2, s_2]$ Train B

- Passing a pole
- Two trains crossing

$$t = \frac{L_1}{s_1}$$

$$t = \frac{L_1 + L_2}{|s_1 - s_2|}$$

$$t = \frac{L_1 + L_2}{s_1 + s_2}$$

Same direction

Opposite direction

Example: A train 120 m long, moving at 54 km/h, crosses another train 180 m long moving at 36 km/h in the opposite direction. Time taken for the two trains to completely cross each other:

- $[L_1 = 120\text{m}, s_1 = 15 \text{ m/s}] \cdot [L_2 = 180\text{m}, s_2 = 10\text{m/s}]$
- Relative speed = $(15 + 10) = 25 \text{ m/s}$
- Distance to cover = $(120 + 180) = 300 \text{ m}$
- Time = Distance / relative speed = $300/25 = 12 \text{ seconds}$

NOTES



Free daily 15-minute GMAT challenge

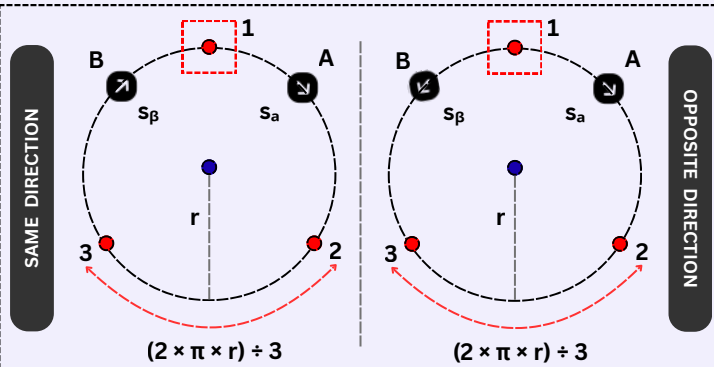
Build consistent study habits at **zero cost**

Take Daily Target



Circular Tracks

Track Length (L) = $2 \times \pi \times r$



NOTE: Points 1, 2, 3 mark successive relative positions where meetings can occur

- Time to first meeting
 $T = \frac{L}{|s_a - s_b|}$
- Meeting point (distance d from start) = $T \times s_a$
- Number of meeting points = $|s_a - s_b|$

- Time to first meeting
 $T = \frac{L}{s_a + s_b}$
- Meeting point (distance d from start) = $T \times s_a$
- Number of meeting points = $s_a + s_b$

✎ If A takes t_a sec and B takes t_b sec for one lap, both meet at start again after **LCM(t_a , t_b)** sec.

✎ **Example:** Two runners A and B run on a circular track of length 600 m in the same direction. The ratio of their speeds is 5 : 3. If they meet for the second time after 450 seconds, find the speed of the faster runner.

- Second meeting $\Rightarrow$ faster runner has gained 2 full laps over the slower runner = $2 \times 600 = 1200$ m
- Relative speed = $(5k - 3k) = 2k \Rightarrow 2k \times 450 = 1200 \Rightarrow k = 4/3$
- Faster runner $\rightarrow 5k = 5 \times (4/3) = 20/3$ m/s

Escalators [Rare]

Effective Speed (s) = Person's Speed (p) $\pm$ Escalator Speed (e)

Time (t) = Total Steps (N) $\div$ Effective Speed (s)

- (+) when person moves in direction of escalator
- (-) when person moves against direction of escalator

Boats

Boat speed $\rightarrow B$ • Stream speed $\rightarrow W$

• Downstream speed (D) = $(B + W) \rightarrow$ moving in stream's direction

• Upstream speed (U) = $(B - W) \rightarrow$ such that $B > W$

✎ If downstream and upstream speeds are known:

$$B = \frac{D + U}{2} \quad W = \frac{D - U}{2}$$

✎ I: Same distance travelled $\rightarrow$ distance = L

✎ Time taken Downstream vs Upstream

$$T_{down} = \frac{L}{B + W} \quad T_{up} = \frac{L}{B - W} \quad \frac{T_{up}}{T_{down}} = \frac{(B + W)}{(B - W)}$$

✎ Difference vs Sum of Time taken

$$T_{up} - T_{down} = \frac{2 \times L \times W}{(B^2 - W^2)} \quad T_{up} + T_{down} = \frac{2 \times L \times B}{(B^2 - W^2)}$$

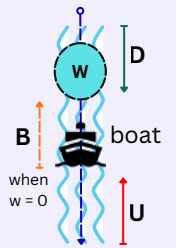
✎ II: Different distance travelled $\rightarrow$ distance = L_1, L_2

✎ If L_1 , downstream and L_2 upstream:

$$T = \frac{L_1}{B + W} + \frac{L_2}{B - W}$$

✎ If upstream and downstream times are given and the time taken in still water needs to be found then, use:

$$\frac{1}{T_{still}} = \frac{1}{2} \left(\frac{1}{T_{up}} + \frac{1}{T_{down}} \right)$$



✎ **Example:** A boat travels a certain distance downstream in 3 hours and returns upstream along the same route in 5 hours. If the speed of the stream is 2 km/h, what is the speed of the boat in still water?

- Boat speed in still water = B
- Stream speed = $W = 2$ kmph
- Downstream speed = $(B + 2)$
- Upstream speed = $(B - 2)$

Distance = Downstream speed $\times$ time taken
 $\Rightarrow (B + 2) \times 3 = 30$ kms

Since distance is the same both ways:

$$3 \times (B + 2) = 5 \times (B - 2) \Rightarrow 3B + 6 = 5B - 10 \Rightarrow 2B = 16 \Rightarrow B = 8 \text{ kmph}$$

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NOTES



ALGEBRA | COUNTING & PROBABILITY | STATISTICS

A EXPONENTS & ROOTS | LOG

INDICES

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$ (where $a \neq 0$)
- $a^{-m} = 1/a^m$
- $(ab)^m = a^m \times b^m$
- $(a/b)^m = a^m / b^m$
- $1^m = 1$
- $(-1)^m = 1$ ($m \rightarrow$ even)
or -1 ($m \rightarrow$ odd)
- $a^1 = a$

Important concepts

- Exponential Equations**
 - If $a^m = a^n$, then $m = n$ (for $a \neq 0, 1, -1$)
 - If $a^m = b^m$, then $a = b$ (for $m \neq 0, m = \text{odd}$)
- Power Comparisons**
 - For $a > 1$: if $m > n$, then $a^m > a^n$
 - For $0 < a < 1$: if $m > n$, then $a^m < a^n$

SURDS

- $\sqrt[n]{a \times b} = \sqrt[n]{a} \times \sqrt[n]{b}$
- $\sqrt[n]{a/b} = \sqrt[n]{a} / \sqrt[n]{b}$ (where $b \neq 0$)
- $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$
- $(\sqrt{a} + \sqrt{b})^2 = a + b + 2\sqrt{ab}$

- $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$
- $\sqrt[n]{a/b} = \sqrt[n]{a} / \sqrt[n]{b}$
- $(\sqrt[n]{a})^m = \sqrt[n]{a^m} = a^{m/n}$
- $\sqrt[n]{m} \sqrt[n]{a} = \sqrt[n]{ma}$
- $a^{(1/n)} = \sqrt[n]{a}$

- $\sqrt{(a^2b)} = a\sqrt{b}$ (where $a \geq 0$)
- $\sqrt{a/b} = \sqrt{a}/\sqrt{b}$
- $1/(\sqrt{a} + \sqrt{b}) = (\sqrt{a} - \sqrt{b})/(a - b)$
- $1/(\sqrt{a} - \sqrt{b}) = (\sqrt{a} + \sqrt{b})/(a - b)$
- $1/(a + \sqrt{b}) = (a - \sqrt{b})/(a^2 - b)$

3. Roots Comparison

- For $a > 0, b > 0$: if $a > b$, then $\sqrt[n]{a} > \sqrt[n]{b}$ (for $n \geq 1$)

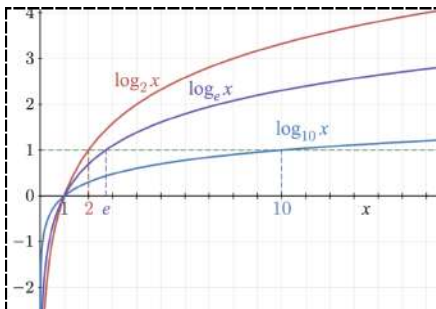
4. Useful Approximations

- $\sqrt{2} \approx 1.414$
- $\sqrt{3} \approx 1.732$
- $\sqrt{5} \approx 2.236$

LOGARITHMS

- If $p^x = q$, then:
• $\log_p q = x$
where:
→ $p > 0$ and $p \neq 1$
→ $q > 0$

- $\log 2 \approx 0.301$
- $\log 3 \approx 0.477$
- $\log 5 \approx 0.699$
- $\log 7 \approx 0.845$



Fundamental Laws

- $\log_a mn = \log_a m + \log_a n$
- $\log_a (m/n) = \log_a m - \log_a n$
- $\log_a m^n = n \times \log_a m$
- $\log_a m = \log_x m / \log_x a$
- $\log_a m = 1 / \log_m a$
- $\log_a m = \log m / \log a$
- $\log_m a = 1 / \log_a m$

Identities

- $\log_a a = 1$
- $\log_a 1 = 0$
- $a^{\log_a x} = x$
- $\log_a a^k = k$
- $\log_a (1/x) = -\log_a x$
- $\ln e^x = x \rightarrow (e \approx 2.718)$
- $e^{\ln x} = x$

Useful for Equations

- If $\log_a m = \log_a n$, then $m = n$
- If $\log_a m > \log_a n$, then:
 - For $a > 1$: $m > n$
 - For $0 < a < 1$: $m < n$
- For $0 < a < 1$:
 - If $x > 1$, then $\log_a x < 0$
 - If $0 < x < 1$, then $\log_a x > 0$
 - If $x = 1$, then $\log_a x = 0$

→ For $a > 1$:

- If $x > y > 0$, then $\log_a x > \log_a y$
- $\log_a x > 0 \Leftrightarrow x > 1$
- $\log_a x < 0 \Leftrightarrow 0 < x < 1$
- For $0 < a < 1$:
 - If $x > y > 0$, then $\log_a x < \log_a y$
 - $\log_a x > 0 \Leftrightarrow 0 < x < 1$
 - $\log_a x < 0 \Leftrightarrow x > 1$

B EQUATIONS

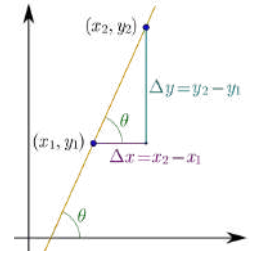
Linear Equations

Slope (Gradient)

- $m = (y_2 - y_1)/(x_2 - x_1)$
- **Horizontal line:** $m = 0$
→ (equation: $y = k$)
- **Vertical line:** $m = \text{undefined}$
→ (equation: $x = k$)
- **Parallel lines:** $m_1 = m_2$
- **Perpendicular lines:** $m_1 \times m_2 = -1$ or $m_2 = -1/m_1$

Approach

- **x-intercept:**
→ Set $y = 0$, solve for x
- **y-intercept:**
→ Set $x = 0$, solve for y
- **Line through origin:**
→ $y = mx$ ($b = 0$)
- **Finding equation from two points:**
 - Find $m = (y_2 - y_1)/(x_2 - x_1)$
 - Use $y - y_1 = m(x - x_1)$



Slope-Intercept Form: $y = mx + b$

→ where m = slope, b = y-intercept

Point-Slope Form: $y - y_1 = m(x - x_1)$

→ m = slope, (x_1, y_1) = point on line

Standard Form: $Ax + By = C$

→ where A, B, C are integers, $A \geq 0$

Intercept Form: $x/a + y/b = 1$

→ where a = x-intercept, b = y-intercept

Quadratic Equations

General Form:

- $ax^2 + bx + c = 0 \rightarrow a \neq 0$
- $y = a(x - r_1)(x - r_2) \rightarrow r_1, r_2 = \text{roots}$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

Important concepts

- Sum of roots: $r_1 + r_2 = -b/a$
- Product of roots: $r_1 \times r_2 = c/a$

⚠ For $ax^2 + bx + c$, the **minimum** (if $a > 0$) or **maximum** (if $a < 0$) occurs at:

$$x = -b/(2a)$$

And the value at that point is:

$$\text{min/max value} = c - b^2/(4a)$$

- **$D > 0$:** Two distinct real roots
- **$D = 0$:** One real root
- **$D < 0$:** Two complex roots
- **Perfect square trinomial:**
 - $x^2 + 2mx + m^2 = (x + m)^2$
 - $x^2 - 2mx + m^2 = (x - m)^2$
- **Difference of squares:**
 - $x^2 - m^2 = (x + m)(x - m)$

Approach

Most equations can be reduced to the following form:

$$a_1x + b_1y = c_1 \quad a_2x + b_2y = c_2$$

Solving by Elimination:

- Multiply equations to get same coefficient for one variable
- Add/subtract equations to eliminate that variable

Possible Outcomes

1. One unique solution:

- Lines intersect at one point
- Condition: $a_1/b_1 \neq a_2/b_2$
- (Slopes are different: $m_1 \neq m_2$)

2. Infinitely many solutions:

- Lines are identical (coincident)
- Condition: $a_1/a_2 = b_1/b_2 = c_1/c_2$

3. No solution:

- Lines are parallel (never intersect)
- Condition: $a_1/a_2 = b_1/b_2 \neq c_1/c_2$

NOTES



Free daily 15-minute GMAT challenge

Take Daily Target

1 If $a > b$, then:

- for any real c , $a + c > b + c$
- $c > 0$, then $ac > bc$
- $c < 0$, then $ac < bc$
- $c > 0$, then $a/c > b/c$
- $c < 0$, then $a/c < b/c$

→ Transitive Property

- If $a > b$ and $b > c$, then $a > c$

→ Chaining Inequalities

- If $a > b$ and $c > d$, then $a + c > b + d$

⚠ Note: $a > b$ and $c > d$ does NOT guarantee $a - c > b - d$

2 Absolute Value

- $|x| < a \Leftrightarrow -a < x < a \rightarrow (a > 0)$
- $|x| \leq a \Leftrightarrow -a \leq x \leq a$
- $|x| > a \Leftrightarrow x < -a$ or $x > a$
- $|x| \geq a \Leftrightarrow x \leq -a$ or $x \geq a$
- $|x - a| < b \Leftrightarrow a - b < x < a + b$
- $|x - a| > b \Leftrightarrow x < a - b$ or $x > a + b$
- $|a \times b| = |a| \times |b|$
- $|a/b| = |a| / |b|$ (if $b \neq 0$)
- $|a + b| \leq |a| + |b|$
- $||a| - |b|| \leq |a - b| \leq |a| + |b|$

3 Mean Inequalities

→ For $a, b > 0$:

- **AM** (Arithmetic Mean): $(a + b)/2$
- **GM** (Geometric Mean): $\sqrt{ab}$
- **HM** (Harmonic Mean): $2/(1/a + 1/b) = 2ab/(a + b)$

→ Inequality: **AM $\geq$ GM $\geq$ HM**

- Equality occurs when $a = b$

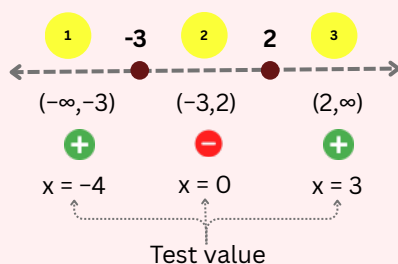
4 For Quadratic Inequalities:

1. Find roots → set expression = 0
2. Plot roots on number line
3. Test each interval → Determine sign pattern

5 Algebraic Identities

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $a^2 - b^2 = (a + b)(a - b)$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
- $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
- $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
- $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
- $(a + b)^2 \geq 4ab$ for all real a, b
- $a^2 + b^2 \geq 2ab$ (since $(a - b)^2 \geq 0$)

Example: $(x - 2)(x + 3) > 0$



If $a^3 + b^3 + c^3 - 3abc = 0$, then either: (1) $a + b + c = 0$ OR (2) $a = b = c$

Common Mistakes

Mistake 1: Squaring both sides carelessly

- Correct: If $a > b \geq 0$, then $a^2 > b^2$
 - Correct: If $0 \geq a > b$, then $a^2 < b^2$
 - Incorrect: If $a > b$ (signs unknown), then $a^2 > b^2$
- Counterexample: $1 > -3$ but $1^2 = 1 < 9 = (-3)^2$

Mistake 2: Multiplying inequalities without sign knowledge

- Allowed: If $a > b$ and $c > d$ and all are positive, then $ac > bd$
 - NOT Allowed: If signs are unknown, cannot multiply inequalities directly
- Counterexample: $3 > 1$ and $-2 > -4$, but $3 \times (-2) = -6 < 1 \times (-4) = -4$

Mistake 3: Incorrect chain subtraction

- WRONG: If $a > b$ and $c > d$, then $a - c > b - d$
- Counterexample: $5 > 4$ and $3 > 1$, then $5 - 3 = 2$ and $4 - 1 = 3$, so $2 < 3 \Rightarrow$ inequality reverses

Mistake 4: Taking reciprocals without considering signs

- Rule: For positive numbers: $a > b > 0 \Rightarrow 1/a < 1/b$
- Rule: For negative numbers: $a < b < 0 \Rightarrow 1/a > 1/b$

Mistake 5: Forgetting the "Or" in absolute value inequalities

- WRONG: $|x| > 4 \Rightarrow x > 4$
- CORRECT: $|x| > 4 \Rightarrow x > 4$ OR $x < -4$

Mistake 6: Assuming variables are positive (+ve)

- Always test: negative, zero, positive fractions, and positive integers.

NOTES

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Start Test Now



1 Arithmetic Progression (AP)

- difference (d) between consecutive terms is constant

General Term

$$a_n = a_1 + (n - 1)d$$

where:

- $d \rightarrow$ common difference
- $a_1 \rightarrow$ first term of the series
- $n \rightarrow$ number of terms
- $a_n \rightarrow$ last term of the series

NOTE: AM = $(a + b)/2$

- If three numbers are in AP:
 $a - b = b - c \Rightarrow 2b = a + c$

Sum of First n Terms

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$$

Alternate formula:

$$S_n = \frac{n}{2}[a_1 + a_n]$$

→ Sum of first n natural numbers:

$$1 + 2 + 3 + \dots + n = \frac{n(n + 1)}{2}$$

→ Sum of squares of first n natural numbers:

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n + 1)(2n + 1)}{6}$$

→ Sum of cubes of first n natural numbers:

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n + 1)}{2} \right]^2$$

→ Sum of first n odd and even numbers:

$$1 + 3 + 5 + \dots + (2n - 1) = n^2 \quad \text{O}$$

$$2 + 4 + 6 + \dots + 2n = n(n + 1) \quad \text{E}$$

◆ Type 2: Geometric series applications

Q: A ball drops from 128m height, rebounds to half its previous height each time. What is the total distance traveled till rest?

⇒ Downward: $128 + 64 + 32 + \dots$ infinite GP

⇒ Upward: $64 + 32 + 16 + \dots$ infinite GP

⇒ Downward sum = $128/(1 - 0.5) = 256\text{m}$

⇒ Upward sum = $64/(1 - 0.5) = 128\text{m}$

∴ **Total** = $256 + 128 = 384\text{m}$

2 Geometric Progression (GP)

- each term is multiplied by a constant ratio (r)

General Term

$$a_n = a_1 \times r^{n-1}$$

where:

- $d \rightarrow$ common difference
- $n \rightarrow$ number of terms

NOTE: GM = $\sqrt{(ab)}$

- If three numbers are in GP: $a/b = b/c \Rightarrow b^2 = ac$

Sum of First n Terms

$$S_n = a_1 \frac{(r^n - 1)}{(r - 1)} \quad \text{OR} \quad a_1 \frac{(1 - r^n)}{(1 - r)}$$

when $r > 1$ when $r < 1$

when $n = (\infty)$ and $|r| < 1$

$$S_\infty = \frac{a_1}{1 - r}$$

3 Harmonic Progression (HP) [less common on GMAT]

- the reciprocal of each term forms an AP

General Term

$$a_n = \frac{1}{a_1 + (n - 1)d}$$

NOTE: HM = $2ab/(a + b)$

- If three numbers are in HP: $1/a - 1/b = 1/b - 1/c \Rightarrow 2/b = 1/a + 1/c$

where: ▪ $d \rightarrow$ common difference ▪ $n \rightarrow$ number of terms

⚠ Triangular Numbers:

$$T_n = 1 + 2 + 3 + \dots + n = n(n + 1)/2$$

Sequence: 1, 3, 6, 10, 15, 21, ...

⚠ Fibonacci-type Sequences:

$$F_n = F_{n-1} + F_{n-2} \text{ with } F_1 = 1, F_2 = 1$$

Sequence: 1, 1, 2, 3, 5, 8, 13, ...

Typical Question Formats

◆ Type 1: Summation with conditions

Q: If the sum of first 20 terms of an AP is 420, and the sum of first 30 terms is 930, then find 25th term.

$$\Rightarrow S_{20} = 20/2 \times [2a + 19d] = 420 \Rightarrow 2a + 19d = 42$$

$$\Rightarrow S_{30} = 30/2 \times [2a + 29d] = 930 \Rightarrow 2a + 29d = 62$$

$$\Rightarrow \text{Subtract: } 10d = 20 \Rightarrow d = 2, a = 2$$

$$\therefore a_{25} = a + 24d = 2 + 48 = 50$$

NOTES



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→ **Counting Principle:** If one event can occur in m ways, and a second event can occur in n ways, then both events together can occur in: $m \times n$ ways [Ex: 3 shirts $\times$ 4 pants = 12 outfits.]

→ **Basic Permutation:** Number of ways to arrange r objects from n distinct objects: $P(n, r) = n! / (n-r)!$

♦ **Example:** 3 books from 5: $P(5, 3) = 5! / (5-3)! = 5 \times 4 \times 3 = 60$

→ Permutations with **Repetition Allowed:** If you have n objects and choose r with repetition allowed: n^r arrangements

♦ **Example:** 3-digit code with digits 0-9: $10^3 = 1000$

→ Permutations of **Identical Objects:** If there are n objects with p identical of type 1, q identical of type 2, and r identical of type 3, then the number of distinct arrangements: $n! / (p! \times q! \times r!)$

♦ **Example:** Arrange "MISSISSIPPI": $11! / (4! \times 4! \times 2!) = 34,650$

→ **Basic Combination:** Number of ways to choose r objects from n distinct objects: $C(n, r) = n! / [r! \times (n-r)!]$

♦ **Example:** 3 students from 10: $C(10, 3) = 10! / (3! \times 7!) = 120$

Combination Properties

- $C(n, r) = C(n, n-r)$ ▪ $C(n, 0) = C(n, n) = 1$ ▪ $C(n, 1) = n$
- $C(n, r) = C(n-1, r) + C(n-1, r-1)$ (Pascal's Rule)

→ Choosing **Multiple Groups:** The number of ways to divide n distinct items into **Group 1:** r items, **Group 2:** s items, **Group 3:** remaining t items (where $r + s + t = n$) is:

$$C(n, r) \times C(n-r, s) \times C(t, t) = n! / (r! \times s! \times t!)$$

Circular Arrangements

- Ways to arrange n distinct objects in a circle: $(n-1)!$
- If clockwise/counterclockwise are identical: $(n-1)!/2$

Arrangements with Restrictions

- **Fixed Positions:** k specific items in fixed positions, arrange remaining $(n-k)$ items: $(n-k)!$ ways
- **Items That Must Be Together:** Treat the group as 1 "super item": $(m+1)! \times k!$ (where m = other items, k = items in the group)
- **Items That Must Not Be Together:** Total arrangements - arrangements with them together: $n! - [(n-1)! \times 2!]$

PROBABILITY

General format:

$P(A) = \text{Favourable Outcomes} \div \text{Total Possible Outcomes}$

- Range: $0 \leq P(A) \leq 1$ ▪ Impossible event: $P(A) = 0$

▪ **Complement Rule:** $\rightarrow P(\text{not } A) = 1 - P(A)$

♦ **Example:** If rain probability = 0.3, then no rain = $1 - 0.3 = 0.7$

▪ **Mutually Exclusive Events:** $\rightarrow P(A \text{ or } B) = P(A) + P(B)$

♦ **Example:** Draw a heart or spade from a deck: $13/52 + 13/52 = 26/52 = 1/2$

▪ **Non-Mutually Exclusive Events:** $\rightarrow P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

♦ **Example:** A heart or king: $13/52 + 4/52 - 1/52 = 16/52 = 4/13$

▪ **Independent Events:** $\rightarrow P(A \text{ and } B) = P(A) \times P(B)$

♦ **Example:** Two coin tosses, both heads: $(1/2) \times (1/2) = 1/4$

▪ **Conditional Probability:** $\rightarrow P(A|B) = P(A \text{ and } B) / P(B)$

♦ **Example:** Probability of heart given card is red: $(13/52) \div (26/52) = 1/2$

Special Cases

→ **Handshakes:** Number of handshakes among n people if each shakes once: $C(n, 2) = n \times (n-1) / 2$

→ **Diagonals:** Number of diagonals in an n -sided polygon: $n \times (n-3) / 2$

→ **Path Counting in a Grid:** Number of paths from bottom-left to top-right in $m \times n$ grid: $C(m+n, m)$ or $C(m+n, n)$

♦ **Example:** 3×4 grid: $C(7, 3) = 35$ paths.

Important Outcomes

1. Probability with "At Most" / "At Least"

- $P(\text{at most } k) = P(0) + P(1) + \dots + P(k)$
- $P(\text{at least } k) = P(k) + P(k+1) + \dots + P(n)$ OR $[1 - P(\text{all failures})]$

♦ **Example:** Probability at least one heads in 3 tosses: $1 - (1/2)^3 = 7/8$

2. Odds vs. Probability

- Odds in favor of $A = P(A) : P(\text{not } A)$
- Odds against $A = P(\text{not } A) : P(A)$
- If odds in favor = $m:n$, then $P(A) = m/(m+n)$

3. Binomial Probability (Fixed n , Independent Trials)

- $P(\text{exactly } k \text{ successes}) = C(n, k) \times p^k \times (1-p)^{n-k}$

♦ **Example:** 5 coin tosses, exactly 3 heads: $C(5, 3) \times (1/2)^3 \times (1/2)^2 = 10 \times 1/8 \times 1/4 = 5/16$

4. Geometric Probability (First Success on k -th Trial)

- $P(\text{first success on } k\text{-th trial}) = (1-p)^{k-1} \times p$

♦ **Example:** First heads on 4th toss: $(1/2)^3 \times (1/2) = 1/16$

Selection Without Replacement

Probability of getting exactly k successes in n draws when drawing without replacement from a finite population where:

- Total items: N
- Total successes in population: K
- Total failures in population: $N - K$

$$P = \frac{C(K, k) \times C(N - K, n - k)}{C(N, n)}$$

♦ **Example:** **Committee selection problems** → A company has 8 men and 6 women. If a 5-person committee is chosen randomly, what is the probability it contains exactly 3 women?

- $N = 14$ total people ($8M + 6W$)
- $K = 6$ women (our "successes")
- $n = 5$ people chosen
- $k = 3$ women wanted

$$\Rightarrow P = [C(6, 3) \times C(8, 2)] \div C(14, 5)$$

$$\Rightarrow [20 \times 28] \div 2002$$

$$\Rightarrow 560 \div 2002 = 280 \div 1001 \approx 0.2797 (\approx 28\%)$$

NOTES



Measures of Central Tendency

1. **Mean** (Average) = (Sum of all values) / (Number of values)

- Weighted mean = $(w_1x_1 + w_2x_2 + \dots + w_nx_n) / (w_1 + w_2 + \dots + w_n)$
- If Group A has mean M_a and size n_a , Group B has mean M_b and size n_b , then combined mean = $(n_aM_a + n_bM_b) / (n_a + n_b)$

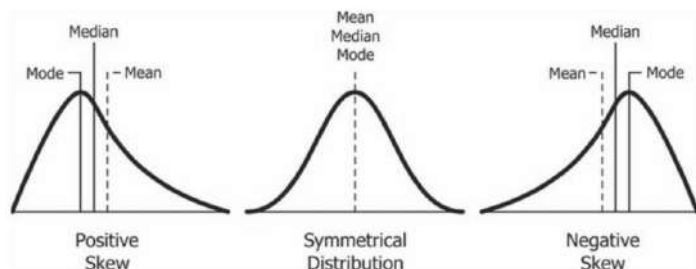
2. **Median** = Middle value when data is arranged in ascending/descending order

- For odd n : Median = Middle term [position $(n+1)/2$]
- For even n : Median = Average of $n/2$ th and $(n/2 + 1)$ th terms

3. **Mode** = Value that appears most frequently

- A set can have no mode, one mode (unimodal), or multiple modes (bimodal, multimodal)

Relationship between Mean-Median-Mode



- For symmetric/ no skew: **Mean** = **Median** = **Mode**
- For skewed right (positive): **Mean** > **Median** > **Mode**
- For skewed left (negative): **Mean** < **Median** < **Mode**

Standard deviation (σ or **SD**) → tells how far the values in the dataset are from the mean.

1. **Clustering:**

- Small σ :** Data clustered near mean
- Large σ :** Data spread out from mean
- Same mean, different σ :** Flatter curve = larger σ

2. **Empirical Rule** (in Normal Distribution):

- Within 1 SD of mean: $\approx 68\%$ of data
- Within 2 SD of mean: $\approx 95\%$ of data
- Within 3 SD of mean: $\approx 99.7\%$ of data

3. **Adding or Subtracting Constant:** If each value $\pm k$, mean changes by $\pm k$, SD unchanged

4. **Multiplying or Dividing by Constant:** If each value $\times k$, mean and SD both multiplied by $|k|$

GMAT Trap: SD is always ≥ 0 . If SD = 0, all values are identical.

Range = Max value - Min value → used for quick spread comparison (often tested with outliers)

Limitation:

- Depends only on extremes
- Easily distorted by one value

Example: {2, 3, 4, 5, 5, 8, 9, 15} → Range = 13

Strategies for Data Sufficiency

- For **Mean** questions → need either:
 - Sum and number of terms, OR
 - Relationship allowing calculation
- For **Median** questions → need exact position and values around it
- For **SD** questions (often tested conceptually) → remember:
 - Adding same number **k** to all terms → SD unchanged
 - Multiplying all terms by **k** → SD multiplied by **|k|**

NOTES



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